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Estimation of Power Generation from Thermoelectric Devices: Model Analysis and Performance Measurements

M. W. Oh,[1,*] J. H. Ahn,[2] J. K. Lee,[3] B. S. Kim,[1] S. D. Park,[1] B. K. Min,[1] Y. S. Choi,[4] and H. W. Lee[1]

[1]Advanced Materials and Applications Research Division, Korea Electrotechnology Research Institute,
28-1 Seongju-dong, Changwon-si, Gyeongnam 641-120, Korea
[2]School of Nano and Advanced Materials Engineering,Gyeongsang National University,
Gajwa-dong, Jinju-si, Gyeongnam 660-701, Korea
[3]Division of Nano-Advanced Materials Engineering, Changwon National University,
Sarim-dong, Changwon-si, Gyeongnam 641-773, Korea
[4]Agency for Defense Development, Daejeon 305-600, Korea

A realistic model for the electrical output power from a thermoelectric device has been developed. The electrical contact resistivity, the thermal contact conductivity, and the thickness of an insulating layer and an electrode were considered to enhance the theoretical model. The maximum output power was achieved in the specific length of a thermoelement, which was largely dependent on the contact properties and the thickness of the insulating layer and the electrode.The measured output power from the fabricated thermoelectric device was understood and compared with the developed model. It was also seen that the derived model can be used to estimate the output power as a function of the dimension of the thermoelectric device. The optimized design was essential to enhance the performance of the thermoelectric device for waste heat recovery.

Keywords: thermoelectric device, thermoelectric power generation, thermoelectric module, thermoelectric properties

1. INTRODUCTION

Thermoelectric power generation has attracted much interest due to its applicability to recover waste heat to energy.[1,2] The advantage of the thermoelectric power is its compactness, zero green-gas usage, and longlifetime, whereas the low energy conversion efficiency is main weak point for the thermoelectric power generation.[3,4] The energy conversion efficiency of the thermoelectric power generation is directly related to the temperature environment and the dimensionless figure of merit, $ZT=\alpha^2T/\rho k$, where α is the Seebeck coefficient, ρ is the electrical resistivity, κ is the thermal conductivity, and T is temperature in Kelvin.[1] Researches in a thermoelectric field had been mostly conducted to find materials showing better *ZT*. However, the power generation is produced from the thermoelectric devices consisted of the p-type and the n-type thermoelectric materials. Thus it is also important to investigate the performance of the thermoelectric device as a function of its shape or design effect, manufacturing factor, and reliability.[6] Among the many aspects influencing the performance, it is known that the shape dimension of the thermoelectric device crucially affects the performance.[5-8] Many theoretical works were reported to elucidate the effect of the shape on the performance, whereas the effect was rarely proved by the experimental works.[5-8] G. Min *et al.* showed the power output of thermoelectric devices was improved by optimizing the length of the thermoelectric element.[5] H. S. Han *et al*. also reported the effect of the dimension of the devices on the output performance and developed the modified model including the temperature dependence of the electrical properties.[7] However most of the experimental works were performed by using conventional thermoelectric device. As a result, it is hard to know the thermoelectric properties of the thermoelectric materials consisting of the devices, resulting in the difficulty to confirm the theoretical models. In this research, the thermoelectric materials were produced and then utilized to fabricate the thermoelectric devices. The dimension of the fabricated devices was varied to see the effect on the performance. The generated power from the thermoelectric devices was measured and the results were compared with the theoretical model.

2. EXPERIMENTAL PROCEDURES

Thermoelectric materials used in fabricating the devices were produced by the conventional melting process. The p-type and the n-type materials are the compounds of Bi-Sb-Te

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*Corresponding author: minwookoh@keri.re.kr

and Pb-Te-Ag-Sb, respectively. These materials were chosen because the thermoelectric properties of them were consistently maintained in all ingots made by ourprocedure. It is thought that the specific melting process and the composition are not important to investigate the shape effect on the output power. Thus the details of the melting process are not presented here. However, the thermoelectric properties of the *p*- and the *n*-type materials are more necessary and important. The results are listed in Table 1. The electrical resistivity and the Seebeck coefficient were measured by using ZEM-3 (ULVAC-RIKO, Japan). The thermal diffusivity was measured by the laser flash method (NETZSCH, LFA-457). A differential scanning calorimeter (NETZSCH, DSC 404C) was used for measurement of the heat capacity. The densities of the samples were measured by the Archimedes method. The thermal conductivity was calculated from the results of the density (d), the heat capacitiy (C_p), and the thermal diffusivity (λ), using the equation: $\kappa = \lambda C_p d$.

Thermoelectric materials were cut to the rectangular pieces (area = 4×4 mm^2) for fabricating the thermoelectric devices. The values of the length of the thermoelectric materials were varied in the range from 1 mm to 10 mm, in order to investigate the effect of the length on the output power of the devices. Thermoelements, namely thermoelectric materials cut into rectangular shape to be used in fabricating the devices, were directly bonded to the copper electrodes with a Sn-based conventional solder. The copper electrodes were bonded on alumina plates by the diffusion bonding in a reducing atmosphere.

The experimental setup for measuring the output power of the thermoelectric devices is composed of a ceramic heater, a copper block for heat sink, and a data acquisition system. The thermoelectric devices were placed between the heater and the copper block. Cooling water was flowed through the copper block to maintain the temperature of the block. Thermal grease was painted at the interface of the heater, the device, and the block to reduce the thermal contact resistance (MOMENTIVE YG6111). The heater and the copper block were pressed with the pressure of 24.5 MPa to reduce the thermal contact resistance. The output power was measured by a digital multimeter (Agilent 34970A). K-type thermocouples were inserted into the holes placed in the heater and the copper block to measure the temperature difference. The external load resistance was used to obtain the output power and ranging between 1 mΩ and 20 Ω. After the temperature difference between the heater and the copper block was stabilized, the open-circuit voltage and close-circuit voltage were measured at every 10 sec. until the voltage deviation was converged within 2%.

Table 1.Thermoelectric properties of *p*-type and n-type thermoelectric materials used in the fabrication of thermoelectric devices. The Seebeck coefficient (α), the electrical resistivity (ρ), and the thermal conductivity (κ) are the values measured at 323 K.

	α (μV/K)	ρ (10^{-6} Ωm)	κ (W/mK)
p-type	106	4.96	1.82
n-type	-322	526	1.09

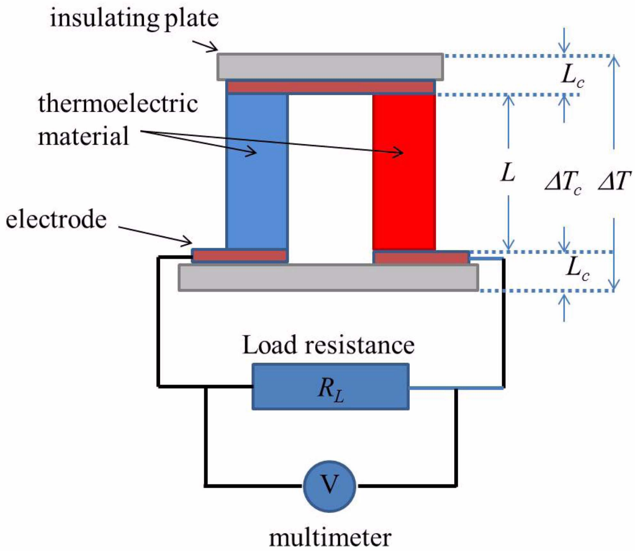

Fig. 1. Schematic diagram of the fabricated thermoelectric device.

3. RESULTS AND DISCUSSION

Schematic feature of the fabricated device is shown in Fig. 1. The unicouple-thermoelectric device was fabricated and its output power was measured. Thermoelectric power generated from the device can be theoretically estimated. The output power is given by

$$P = I^2R_L = \frac{((\alpha_p - \alpha_n)\Delta T)^2}{(R_L + R_i)^2}R_L, \tag{1}$$

where P is the output power, I is the current, R_i is the internal resistance of the device and the lead wires, R_L is the external load resistance, and ΔT is the temperature. α_p and α_n is the Seebeck coefficient of the p-type and the n-type thermoelements, respectively.[5] For simplifying, the absolute value of both p-type and n-type Seebeck coefficient can be considered as same: $\alpha = |\alpha_p| = |\alpha_n|$. Eq. 1 is now given by

$$P = \frac{4(\alpha\Delta T)^2}{(R_L + R_i)^2}R_L. \tag{2}$$

In Eq. 2, it can be easily known that the output power is dependent on R_L. In order to maximize the output power, R_L should be optimized. At optimized R_L, the following equation is satisfied,

$$\frac{\partial P}{\partial R_L} = 0 \tag{3}$$

which yields the equation as follows,

$$4(\alpha\Delta T)^2(R_L+R_i)^2-4(\alpha\Delta T)^2R_L(2R_L+2R_i)=0\,. \tag{4}$$

From Eq. 4, R_L=R_i is obtained. It is noticeable that R_L should be identical to R_i to maximize the output power. When R_L is matched with the internal resistance, the maximum output power is given by,

$$P_{\max}=\frac{(\alpha\Delta T)^2}{R_L}=\frac{(\alpha\Delta T)^2}{R_i}\,. \tag{5}$$

Eq. 5 indicates that maximum output power is only dependent on the temperature difference, the Seebeck coefficient, and the total electrical resistance of a thermocouple. Assuming temperature difference can be sustained regardless the dimension of the device, the output power is varied as a function of the dimension of the thermoelement and given by,

$$P_{\max}=\frac{(\alpha\Delta T)^2}{\rho_p(L_p/A_p)+\rho_n(L_n/A_n)}\,, \tag{6}$$

where L_p and L_n is the length of the p-type and the n-type thermoelements, respectively. A_p and A_n is the cross sectional area of the p-type and the n-type thermoelements, respectively. In practical, the length of the thermoelements are design to be equal:L=L_p=L_n. To simplify model, A_p can be controlled as identical to A_n:A=A_p=A_n. Thus Eq. 6 is simplified as follows,

$$P_{\max}=\frac{(\alpha\Delta T)^2A}{L(\rho_p+\rho_n)}\,. \tag{7}$$

Eq. 7 indicates that the output power is increased as the length of the thermoelements is decreased and approaches infinity as the length goes to zero. This behavior is unrealistic in the real situation. This problem arises from the neglect of the electrical and the thermal contact resistance. As the length of the thermoelements is decreased, the effect of the contact resistance on the internal resistance is increased.

When the electrical contact resistance between the electrodes and the thermoelements is considered, the total internal resistance with the contact resistance (R_{ic}) is given by,

$$R_{ic}=R_i+R_c=\frac{L}{A}(\rho_p+\rho_n)+\frac{2}{A}(\rho_{pc}+\rho_{nc})\,, \tag{8}$$

where R_c is the total electrical contact resistance, ρ_{pc} (ρ_{nc}) is the electrical contact resistivity at the interface between the p-type (n-type) thermoelement and the electrode.

When the thermal contact conductivity (κ_c) is taken into account, the total thermal conductivity with the contact conductivity (κ_{tc}) can be expressed as,

$$\frac{L_t}{\kappa_{tc}}=\frac{2L_c}{\kappa_c}+\frac{L}{\kappa}, \tag{9}$$

where L_t is total length including the thermoelements, the electrodes, and the insulating plates: L_t=L+2L_c. L_c is the length between the electrode and the insulating plate. k_c is the averaged thermal contact conductivity of the electrode and the insulating plate. κ_c and L_c should be same in both *p*-type and *n*-type legs due to use of the same electrode and the insulating plate, whereas κ should be different in the *p*-type and the *n*-type leg. However, it is assumed that the difference between κ of the *p*-type and the *n*-type is so small that the identical k is used in whole procedure hereafter.

Although the temperature difference between the hot side and the cold side is measured as same value of ΔT, actual temperature difference (ΔT_c) at thermoelement is different from the measured one due to the thermal contact conductivity. Because the heat flow (q) is constant through the leg, q can be expressed as,

$$q=\kappa_{tc}\frac{A}{L_t}\Delta T=\kappa\frac{A}{L}\Delta T_c\,. \tag{10}$$

Using Eqs. 9 and 10, ΔT_c is given by,

$$\Delta T_c=\frac{\kappa_c L}{(L\kappa_c+2L_c\kappa)}\Delta T\,. \tag{11}$$

Finally, substituting R_{ic} and ΔT_c for R_i and ΔT in Eq. 5, the output power with considering the electrical contact resistance and the thermal contact conductivity (P_{cmax}) is given by,

$$P_{c\max}=\frac{\alpha^2\Delta T^2(\kappa_cL)^2}{\frac{L}{A}\left[\rho_p+\rho_n+\frac{2}{L}(\rho_{pc}+\rho_{nc})\right](L\kappa_c+2L_c\kappa)^2}$$

$$=\frac{\alpha^2\Delta T^2(\kappa_cL)^2}{\frac{L(\rho_p+\rho_n)}{A}\left[1+\frac{2(\rho_{pc}+\rho_{nc})}{L(\rho_p+\rho_n)}\right](L\kappa_c+2L_c\kappa)^2}$$

$$\text{or } P_{c\max}=\frac{P_{\max}}{\left(1+\frac{2n}{L}\right)\left(1+2\frac{L_c}{L}\frac{\kappa}{\kappa_c}\right)^2}\,, \tag{12}$$

where $n=(\rho_{pc}+\rho_{nc})/(\rho_p+\rho_n)$. Thus the output power with the electrical and the thermal contact properties is reduced by a factor of $(1+2n/L)(1+2L_c\kappa/L\kappa_c)$. To minimize the reduction of the output power, the smaller electrical contact resistivity and the thickness of the electrode and the insulating plate, and the larger thermal contact conductivity are essential.

The values of the output power without and with the electrical contact resistivity expressed in Eqs. 7 and 12, respectively, are shown in Fig. 2 as a function of the length of the thermoelement. It should be noted that the output power is obtained with the measured thermoelectric properties listed in Table 1. Thus the absolute power and the optimum length are considered as a realistic value which can be experimentally achievable and applicable. As mentioned earlier, the output power without the electricalcontact resistivity are increased as the length is decreased and then approached to

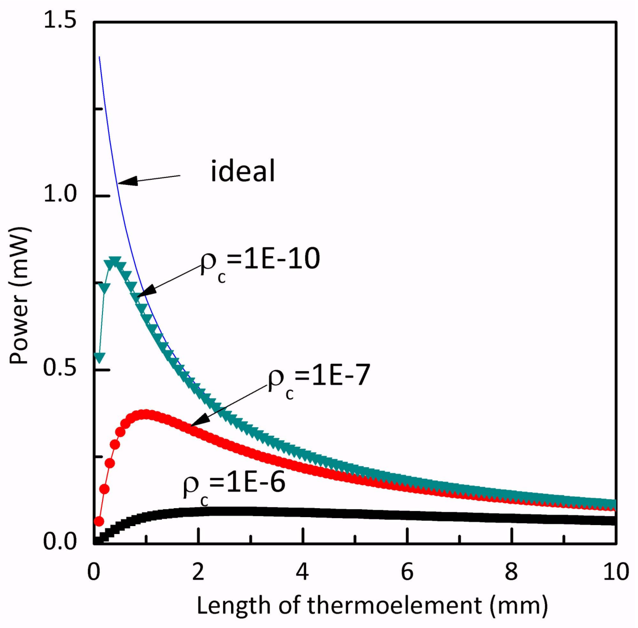

Fig. 2. Thermoelectric output power as a function of the length of the thermoelements for $\kappa_c = 5$ W/mK, $\Delta T = 30$ K and $L_c = 1$ mm. The value of the electrical contact resistivity is varied.

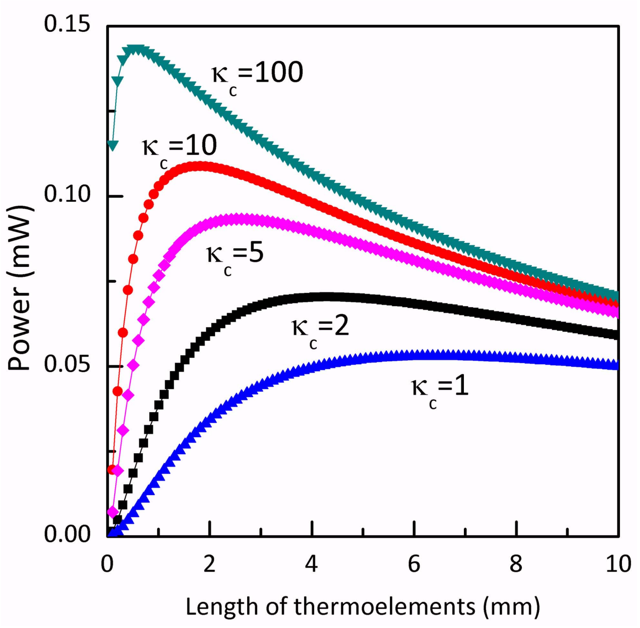

Fig. 3. Thermoelectric output power as a function of the length of the thermoelements for $\rho_{pc} = \rho_{nc} = 10^{-6}\Omega\text{m}^2$, $\Delta T = 30$ K, and $L_c = 1$ mm. The value of the thermal contact conductivity is varied.

infinity as the length goes to zero. However, the output power with the electrical contact resistivity shows the maximum power at the specific length. This implies that the sophisticate designing of the thermoelectric device is crucial for obtaining the maximized output power. The effect of the electrical contact resistivity is mostly eliminated after 10 mm. As can be expected from Eq. 12, the output power is increased with the decreasing electrical contact resistivity.

The effect of the thermal contact conductivity on the output power of the device is plotted in Fig. 3 as a function of the length of the thermoelement. As the thermal contact conductivity is increased, the output power is increased. This is attributed to the smaller temperature reduction in the L_c region with the larger κ_c, which gives the larger temperature difference between the cold and the hot side of the thermoelement. If the value of κ_c is comparable with that of the thermoelement, the length of the thermoelement can be optimized in the broad range, which means the effect of the variation of the length near the optimized point on the output power is negligible. However when the good thermal contact system can be applicable to the device, the length of the thermoelement should be optimized.

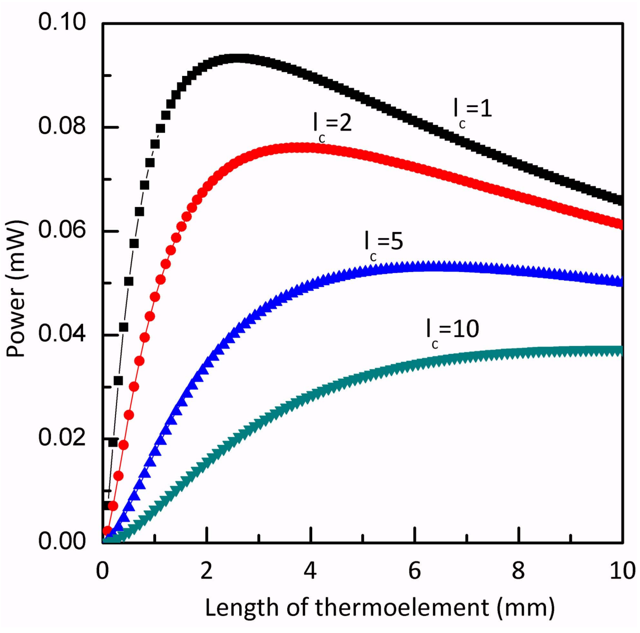

Fig. 4. Thermoelectric output power as a function of the length of the thermoelement for $\kappa_c = 5$ W/mK, $\rho_{pc} = \rho_{nc} = 10^{-6}\Omega\text{m}^2$, and $\Delta T = 30$ K. The value of the thickness of the insulating plate and the electrode is varied.

The effect of L_c on the output power is shown in Fig. 4 as a function the length of the thermoelement. As L_c is increased, the output power is decreased. This can be understood with the reason mentioned in the result of κ_c. The larger L_c presents the larger temperature drop in the region of the electrode and the insulating plate rather than the thermoelement, resulting in smaller temperature difference at both ends of the thermoelements and thus smaller output power. It should be also considered that the length of the thermoelement should be designed with the consideration of the thickness the electrode and the insulating plate.

The effect of the temperature difference between the cold and the hot side of the device on the output power is shown in Fig. 5 as a function of the length of the thermoelement. It is estimated that the output power is increased parabolically with the increase of the temperature difference as expected in Eq. 12. The optimized length is about 2.6 mm for all temperature difference and is not dependent on the temperature.

The experimentally measured output power of the fabricated device is shown in Fig. 6 as a function of the temperature difference. As mention earlier, the output power is parabolically increased as the temperature difference is increased, which is in accordance with the expected theoretically. The output power is varied with the change of the length of the thermoelement. There is no linear relationship between the length and the power, whereas the maximum output power is achieved in the specific length of the thermoelement. This may be due to the effect of the contact properties such as the electrical contact resistivity and the thermal contact resistivity on the output power.

Figure 7 shows the measured output power as a function of the length of the thermoelement. The output power is increased as the temperature difference is increased. The output power is maximized in $L = 2$ mm. It is known previously that this maximum point may be related the value of the electrical contact resistivity, the thermal contact conductivity, and L_c. To specify the more exact length of the thermoelement for enhancing the power, the length is varied in the range of 1 mm to 3 mm. The maximum power is achieved in $L = 1.6$ mm, where the output power is larger by a factor of 1.74 than that of $L = 2$ mm. The optimized length can be also evaluated from Eq. 12. To have $L = 1.6$ for the optimization, it is estimated that κ_c is in the range of 5 to 10 and ρ_c is of 10^{-10} to $10^{-8}\,\Omega m^2$. These values are in the range of the experimentally reported contact properties.[6] Thus the obtained model is seemed to be good enough to evaluate roughly the optimum length of the thermoelement. To compare directly the experimental results with the analyzed model, the measurement of the electrical contact resistivity and the thermal contact conductivity may be needed in future work. There are peculiar points where the value of output power is deviated from the expected tendency profile. This may be due to the incomplete fabrication of the device. It is true that the electrical contact resistivity and the thermal contact conductivity significantly affect the optimized length and these properties are affected by the fabrication procedure.[6] It should be noted that the fabrication procedure is handled as sophisticated as possible to maintain the homogeneity in the properties. However there is difficult to make the excellent bonding between the thermoelement and the electrode in level of the laboratory. Most important thing here is that the dependency of the output power on the length of the thermoelement is clearly seen, which can be roughly evaluated from the obtained equation. To enhance the theoretical estimation, the temperature dependence of the thermoelectric properties and inhomogeneous properties between the *p*-type and the *n*-type thermoelectric materials may be considered.

4. CONCLUSIONS

The thermoelectric devices were fabricated and the output power of them was measured as a function of the length of

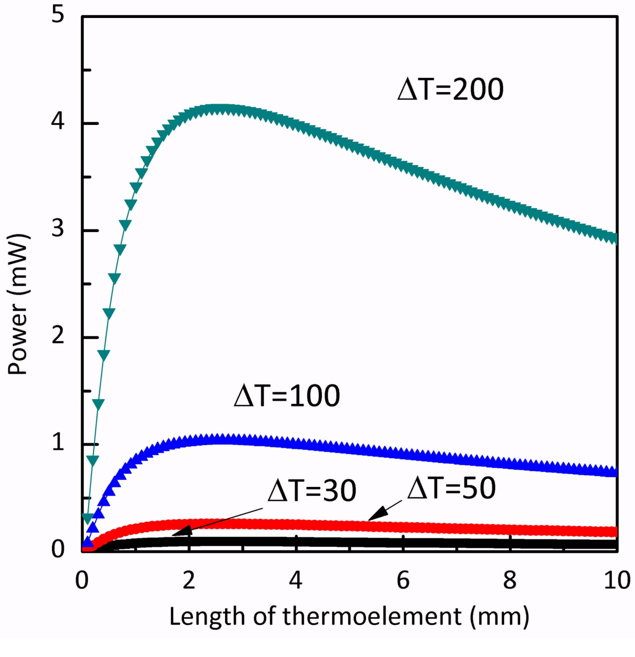

Fig. 5. Thermoelectric output power as a function of the length of the thermoelement for κ_c=5 W/mK, ρ_{pc}=ρ_{nc}=$10^{-6}\Omega m^2$, and L_c=1 mm.The value of the temperature difference is varied.

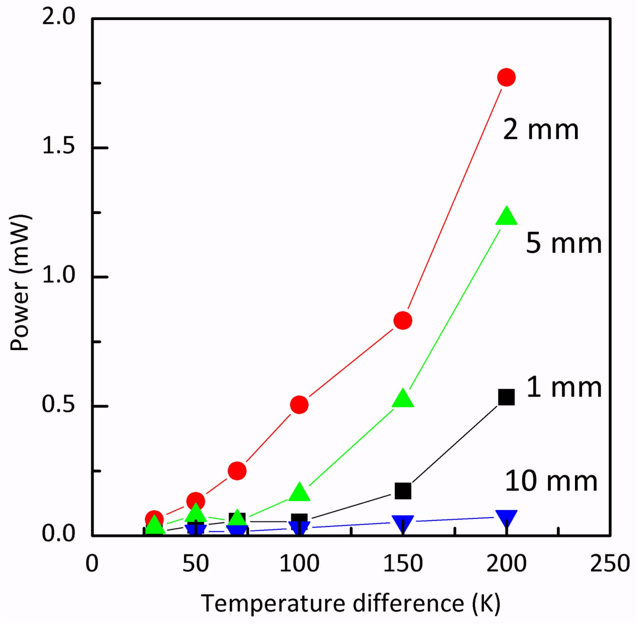

Fig. 6. Measured maximum output power as a function of the temperature difference.

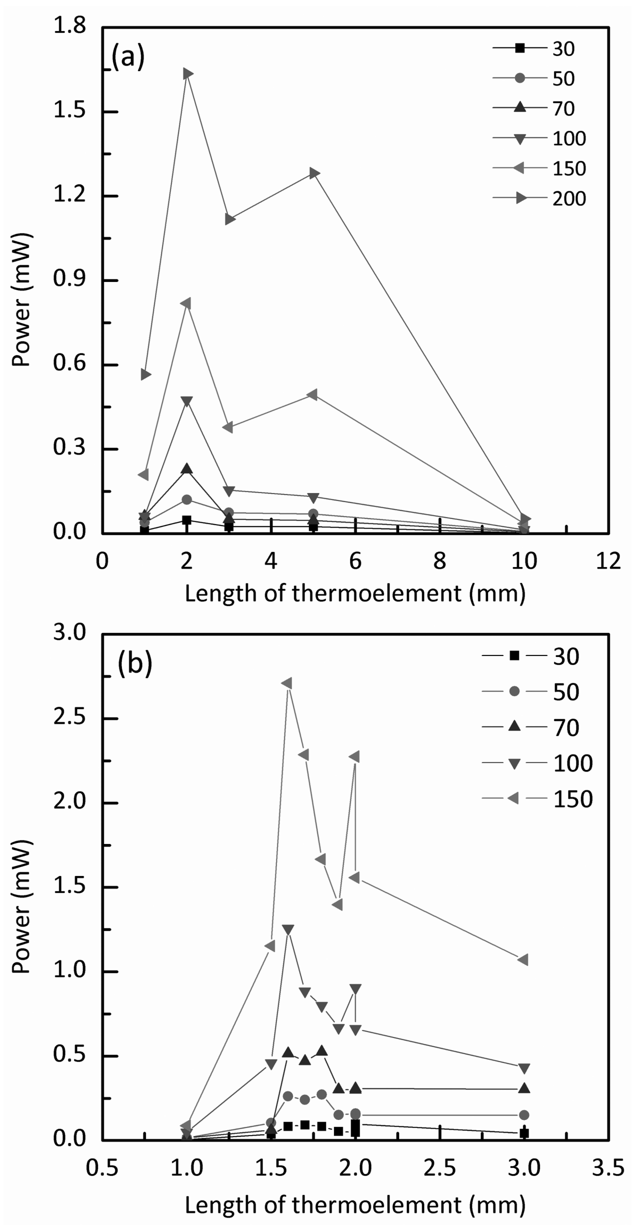

Fig. 7. Measured maximum output power as a function of the length of the thermoelement.(a) 1 mm $< L <$ 10 mm and (b) 1 mm $< L <$ 3 mm.

the thermoelement. The analytical model to estimate output power as a function of the length was developed and the results were compared with the experimental results. It is seen that the maximum power is clearly dependent on the electrical contact resistivity and the thermal contact conductivity. The smaller electrical contact resistivity and the larger thermal contact conductivity are essential to obtain the enhanced output power. The thickness of the insulating plates and the electrodes should be minimized to enhance the output power. The measured output power reveals that the results are significantly affected by the length of the thermoelement and the maximum value is achieved in the optimized length. To fabricate the thermoelectric device which converts the waste heat to the electrical energy as much as possible, the sophisticate design of the device is necessary.

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